MTH 1420, SPRING 2012
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SECTION 5.5: INTEGRATION BY SUBSTITUTION

HW: 6, 12, 18, 26, 31, 34, 42, 47, 63

Practice: 5, 9, 13, 19, 21, 29, 39, 45, 53, 55

1. Introduction

So far we have dealt with antiderivatives only for rather straightforward functions. Finding antiderivatives becomes much more complicated when we are dealing with functions where you must apply the Product or Chain rules. In this section, we look at a method for doing the Chain rule backwards, known as the method of substitution.

Example 1. Let $f(x) = (6x^2 + 1)^5$.

Exercise 2. Try to "think backwards" in the same way as above to find the indefinite integrals below. You can check your answer by taking the derivative.

- $\displaystyle\int 6x \sin(3x^2 - 7)\, dx$

- $\displaystyle\int \frac{(\ln x)^3}{x}\, dx$

2. The Method of Substitution

The big idea is this: $\frac{d}{dx}f(g(x)) =$ __________________________ by the chain rule.

This implies that:

Our goal is to write our integrand in this form somehow. The tricky part is often to find what the "inner" function is.

Method of Substitution	Example : $\int x\sin(3x^2-7)\,dx$
1) Let $u = g(x),$ where $g(x)$ is the "inner function".	1)
2) Find $du = g'(x)dx,$ and solve for dx.	2)
3) Substitute u and du back into the integral, canceling until you have only u's left.	3)
4) Evaluate the integral.	4)
5) Replace u by $g(x)$.	5)

Example 3. Suppose $f'(u) = \dfrac{(\ln u)^3}{u}$ and $f(1) = 2$. Find f.

Exercise 4. Find $\int e^{7x} dx$.

Sometimes it is not initially clear how you can use substitution to take an integral, and you have to mess around with the expression a bit.

Example 5. Calculate $\int \tan x \, dx$.

3. Substitution with Definite Integrals

The method for definite integrals is similar, but we have to be careful with our limits of integration. I will teach you two ways to do it, the bad way and the good way.

The Bad Way: For this method, you act as if you are taking an indefinite integral. Use substitution just like we did before to get an antiderivative (making sure you substitute back in for u!!!!!!) and then evaluate the integral using the limits of integration.

Example 6. $\displaystyle\int_0^1 \frac{e^z + 1}{e^z + z}\, dz$

The Good Way: Here you leave the limits of integration on the integral, but when you substitute in the u, you change the limits of integrationto be for u, not x! Then solve that definite integral as normal.

Example 7. $\displaystyle\int_0^1 \frac{e^z + 1}{e^z + z}\, dz$

Exercise 8. Find $\int_0^7 \sqrt{4+3x}\,dx$.

Sometimes the substitution is not even remotely straightforward.

Example 9. Find $\int_{-1}^1 v^2(1-v)^6\,dx$.

Exercise 10. Find $\int_0^1 30x(e^{3x^2+7})^5\,dx$.

Exercise 11. Find $\int_{-\pi/6}^{\pi/6} \tan^3\theta\,d\theta$.